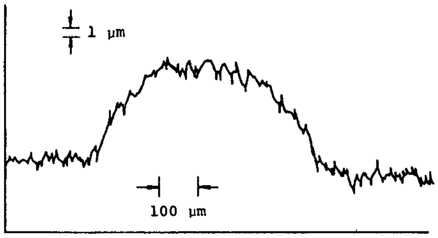

Fig. 7. Film condition of gold thick film utilizing chemical bonding.

it can be noted that thick-film transmission loss is close in value to that of thin film.

This study proves that thick film can be fully utilized up to a frequency of 10 GHz, and it can be concluded that it will be used in low-cost MIC's in mass-produced equipment. Thick film which uses copper (its characteristic impedance is 50 Ω and the substrate thickness is 0.635 mm) has a low transmission loss: 0.005 dB/cm at 0.2 GHz, 0.027 dB/cm at 2 GHz, 0.050 dB/cm at 5 GHz, and 0.087 dB/cm at 10 GHz.

Acknowledgment

The authors wish to express their thanks to Dr. Seki, Dr. Kanmuri, and S. Urabe for their able guidance and encouragement.

References

[1] J. Ramy, "Optimization of the thick- and thin-film technologies for microwave circuits on Alumina and fused silica substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 814–820, Oct. 1978.
[2] G. L. Matthaei, L. Young, and E. M. T. Jomes, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*. New York: Mc-Graw-Hill, p. 206.
[3] R. A. Pucel, D. J. Masse, and C. P. Hartwig, "Loss in microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, no. 6, pp. 342–350.
[4] J. D. Welch and H. J. Pratt, "Losses in microstrip transmission systems for integrated microwave circuits," NEREM Rec., vol. 8, pp. 100–101, 1966.
[5] A. E. Sanderson, "Effect of surface roughness on propagation of the TEM mode," *Advances in Microwaves*, vol. 7, L. Young, Ed. New York: Academic Press, 1971.

Taking into Account the Edge Condition in the Problem of Diffraction Waves on Step Discontinuity in Plate Waveguide

V. P. LYAPIN, V. S. MIKHALEVSKY, and G. P. SINYAVSKY

Abstract —**A way of improving the convergence of the field-matching method by means of taking into account the edge conditions is proposed. The unknown function, as defined on the matching lines, should be expanded into a infinite set of the orthogonal Gegenbauer polynomials having the required singularity.**

One of the ways of improving the convergence of the field-matching method (FMM) being used for solving various problems of electrodynamics is taking into account the edge condition in the vicinity of the edge [1], [2]. In the present paper, this technique is applied to the problem of wave diffraction on the step-diaphragm junction in plate waveguides. This is a typical problem to which the calculation of various step discontinuities in rectangular waveguides is reduced.

Manuscript received March 10, 1981; revised March 1, 1982.
The authors are with the Physics Department, Rostov State University, Spachy Ave., 192, Rostov-on-Don 344090, U.S.S.R.

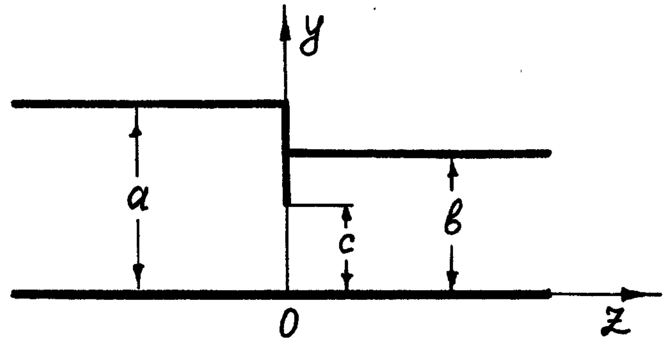

Fig. 1. Step-diaphragm junction in plate waveguide.

Unlike the paper [3] where the edge condition is considered as a correction term to the probe function describing the field in the apperture, in this paper the edge condition is taken into account by every term of the expansion.

Let the TM_p-wave with nonzero field components H_x, E_y, E_z be incident from the wide waveguide onto the obstacle (Fig. 1). This particular problem can be reduced to the solution of an integral equation relative to the unknown distribution function $f(y)=(k\gamma_p)^{-1}E_y|_{z=0}$ for a field component in the diaphragm aperture

$$\int_0^c f(y)\sum_{n=0}^{\infty}\left\{\frac{1}{\gamma_n}\psi_n(y)\psi_n(y')+\frac{1}{\lambda_n}\varphi_n(y)\varphi_n(y')\right\}dy=2\psi_p(y') \tag{1}$$

where

$$\psi_n(y)=\sqrt{\frac{2-\delta_{0n}}{a}}\cos d_n y \qquad \alpha_n=\frac{n\pi}{a} \qquad \gamma_n^2=k^2-d_n^2$$

$$\varphi_n(y)=\sqrt{\frac{2-\delta_{0n}}{b}}\cos\beta_n y \qquad \beta_n=\frac{n\pi}{b} \qquad \lambda_n^2=k^2-\beta_n^2$$

where δ_{0n} is the Kroneker symbol and $k=2\pi/\lambda$.

The approximate solution of this equation can be found by means of Galerkin's method in the form

$$f(y)=\sum_{i=0}^{N}V_iX_i(y) \tag{2}$$

where

$$X_i(y)=\left[1-(y/c)^2\right]^{-1/2}T_{2i}(y/c)$$

and $T_{2i}(y)$ is the Chebyshev polynomials of the first kind.

With the above chosen basis-function set $X_i(y)$, the boundary condition at $y=0$ is satisfied and the function $f(y)$ possesses the required singular behavior when $y\to c$ [4], the infinite set of $\{X_i\}$ is complete and orthogonal on the interval $[0,c]$. The unknown expansion coefficients V_i can be determined from the system of linear algebraic equations

$$\sum_{i=0}^{N}V_iC_{ik}=R_k, \qquad k=0,1,\cdots,N \tag{3}$$

TABLE I
THE DIAPHRAGM SUSCEPTANCE B

N \ M	10	20	30	50	100
0	1.5505	1.5807	1.5910	1.5995	1.6059
1	1.5076	1.5507	1.5649	1.5763	1.5847
2	1.4935	1.5475	1.5636	1.5759	1.5847
3	1.4673	1.5413	1.5610	1.5750	1.5846
4	1.4264	1.5318	1.5570	1.5737	1.5842

where

$$C_{ik} = \sum_{n=0}^{\infty} (2-\delta_{0n}) \left\{ \frac{\phi_i(\alpha_n)\phi_k(\alpha_n)}{a\gamma_n} + \frac{\phi_i(\beta_n)\phi_k(\beta_n)}{b\lambda_n} \right\}$$

$$R_k = 2\sqrt{\frac{2-\delta_{0n}}{a}}\,\phi_k(\alpha_p), \qquad \phi_i(\alpha) = (-1)^i T_{2i}(\alpha_c) \tag{4}$$

and $T_i(\alpha)$ is the Bessel function of the first kind.

As a particular example demonstrating the convergence of the technique discussed, it appears convenient to consider the problem of diffraction of the TM_0-wave by a diaphragm with dimensions $2c = a = b$, as the exact solution for this special case has been already obtained by the Wiener–Hopf technique [5].

In Table I are presented the values of the imaginary part B of the normalized input susceptance Y for $a = 0.4\lambda$ and various N and M (M is a maximum value of the summation index in (4)). The B values are determined by the reflection coefficient A_0 of an incident wave by the ratio

$$Y = G + jB = \frac{1-A_0}{1+A_0} \qquad A_0 = \frac{V_0 R_0}{2\gamma_0} - 1.$$

The exact value of Y calculated in [5] is $Y = 1 + 1.5931j$.

The presented results enable us to draw the following conclusions.

1) The convergence of the solution, depending on the number of functions needed to approximate the field behavior at the boundary of matching, is much better here as compared with the conventional FMM. In fact, the error of the first-order approximation ($N = 0$) does not exceed one per cent here, whereas it equals about 40 percent for a conventional method [6].

2) The ratio $M/N = a/c$ found in [6] as a condition, which provides the best accuracy of the result in conventional FMM technique, does not work when the discontinuity is taken into account by the method presented here. The correct results in the given case can be obtained only under $M \gg N$ conditions. When the ratio M/N is decreasing, the deterioration of convergence occurs due to insufficient calculation accuracy of the matrix elements.

An attempt to improve the accuracy by means of a direct increase of the number of terms included in the sums (4) results in the increase of computing time. Calculation can be made reasonable by using, for calculating the sum terms with large indices, only the first term of their asymptotic expansion. Then the formula for C_{ik} takes the form

$$C_{ik} \simeq \sum_{n=0}^{M} (2-\delta_{0n}) \left[\frac{\phi_i(\alpha_n)\phi_k(\alpha_n)}{a\gamma_n} + \frac{\phi_i(\beta_n)\phi_k(\beta_n)}{b\lambda_n} \right] + S$$

$$S = -j\frac{2}{\pi^2 c} \sum_{n=M+1}^{\infty} \frac{1}{n^2}\left[a(1+\sin 2\alpha_n c) + b(1+\sin 2\beta_n c)\right]. \tag{5}$$

TABLE II
THE DIAPHRAGM SUSCEPTANCE B CALCULATED WITH THE CORRECTION S

N \ M	10	20	30	50	100
0	1.6122	1.6123	1.6123	1.6123	1.6123
1	1.5932	1.5932	1.5932	1.5932	1.5932
2	1.5931	1.5931	1.5931	1.5931	1.5931
3	1.5922	1.5931	1.5931	1.5931	1.5931
4	1.5843	1.5929	1.5931	1.5931	1.5931

The S value is the same for all C_{ik} and is independent of frequency, so S needs to be evaluated only once for the given dimensions. The results of calculating the diaphragm susceptance, for the previously mentioned dimensions, as obtained with the help of formula (5) are listed in Table II.

As it follows from the table, such a technique for calculating the C_{ik} with respect to (5) provides a rapid convergence of the approximate solutions to an exact one. The convergence slightly decreases when $c \to b$. In the extreme case when $c = b$, the nature of the singularity is altered [4]; therefore, so too are the functions by which $f(y)$ is expanded, and the following should be used:

$$X_i(y) = \left[1-(y/c)^2\right]^{-1/3} C_{2i}^{1/6}(y/c)$$

where $C_{2i}^{1/6}(y)$ is the Gegenbauer polynomials. Accordingly, ϕ_i and S given in (4) and (5) also change, thus

$$\phi_i(\alpha) = (-1)^i \alpha^{-1/6} T_{2i+1/6}(\alpha b)$$

$$S = -j\pi^{-31/3}\left\{ \frac{4a^{4/3}}{b} \sum_{n=M+1}^{\infty} \frac{\cos^2(\alpha_n b - \pi/3)}{n^{21/3}} + b^{1/3} \sum_{n=M+1}^{\infty} \frac{1}{n^{21/3}} \right\}. \tag{6}$$

As the carried-out calculations revealed, the convergence of the results in the case of a step junction, without a diaphragm, of two waveguides with unequal heights is similar to the convergence for the structure discussed above. For making calculations, it is sufficient to take into account in the expansion of (2) only 2–3 terms and the M value may equal 10–20.

The solution of the problem of the TE-wave diffraction by a step-diaphragm junction is identical. It was found that the convergence of results in this case was similar to that for the above examples. Thus, the developed method provides a rapid convergence of the approximate results to the exact one, and therefore permits the reduction of the order of the system of algebraic equations. This method can be employed for calculating thin and thick diaphragms in rectangular waveguides, slow-wave ridge systems, various types of junctions, and displacements of waveguides and similar structures.

REFERENCES

[1] G. F. Zargano, A. M Lerer, V. P. Lyapin, V. S. Mikhalevsky, and G. P. Sinyavsky, *Compound Cross-Section Waveguides*. Rostov University Press, Rostov-on-Don, USSR: Univ. Press, 1979.
[2] G. F. Zargano, A. M Lerer, V. S. Mikhalevsky, and G. P. Sinyavsky, "Partial region method application with edge condition to the problem about ridge and crossed waveguides", *Radiotekhnika i electronika*, vol. 22, pp. 2068–2073, no. 10, 1977.
[3] C Vassalo, "On a direct use of edge condition in modal analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 208–212, Apr. 1976.
[4] R Mittra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves*. New York: Macmillan, 1971.

[5] L. A. Weinstein, "The Theory of Diffraction and the Factorization Method," in *Generalized Wiener–Hopf Technique*. Boulder, CO: Golem, 1969.

[6] S. W. Lee, W. R. Jones, and J. J. Campbell, "Convergence of numerical solutions of iris-type discontinuity problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 528–536, June 1971.

Exact Derivation of the Nonlinear Negative-Resistance Oscillator Pulling Figure

J. OBREGON AND A. P. S. KHANNA

Abstract —**Only approximate relations are so far available for the pulling figure of an oscillator. An exact derivation of the pulling figure is presented here, taking fully into account the nonlinearity of the oscillator admittance. Effect of the oscillator nonlinearity on the asymmetry of the pulling range is presented.**

I. Introduction

Oscillator frequency variation with the load changes is often represented by its pulling figure. The pulling figure has so far been calculated either by neglecting the oscillator admittance variation with the RF voltage [1] or has been calculated by approximately taking into account the transferred admittance in the oscillator plane for a small-load perturbation [2]. We present here an exact derivation of the pulling figure taking fully into account the nonlinear behavior of the oscillator admittance. The relation between the asymmetry of the oscillator pulling range and the nonlinearity of the oscillator admittance has been derived. Pulling figures for certain particular cases are also presented.

II. Frequency Variation with the Load Changes

The oscillation condition at the oscillator-output plane without any load perturbation is represented by

$$Y_{T0} = Y_T + Y_0 = 0 \tag{1}$$

where Y_T is the oscillator nonlinear output admittance and Y_0 is the load admittance.

If the oscillations exist with a load perturbation of ΔY_L in the oscillator output plane and writing $\Delta\omega$ and ΔV as the corresponding frequency and RF voltage changes, the oscillation condition can be represented by

$$Y_{T0} + \Delta Y_L + \frac{dY_{T0}}{d\omega} \cdot \Delta\omega + \frac{dY_{T0}}{dV} \Delta V = 0. \tag{2}$$

From (1) and (2)

$$\Delta Y_L + \frac{dY_{T0}}{d\omega} \cdot \Delta\omega + \frac{dY_{T0}}{dV} \Delta V = 0 \tag{3}$$

separating into real and imaginary parts

$$\Delta G_L + \frac{dG_{T0}}{d\omega} \Delta\omega + \frac{dG_{T0}}{dV} \Delta V = 0 \tag{4}$$

Manuscript received November 18, 1981; revised February 9, 1982.

The authors are with Laboratoire d'Electronique des Microndes, E.R.A. au C.N.R.S., U.E.R. des Sciences, 123 rue Albert-Thomas, 87060 Limoges, Cedex, France.

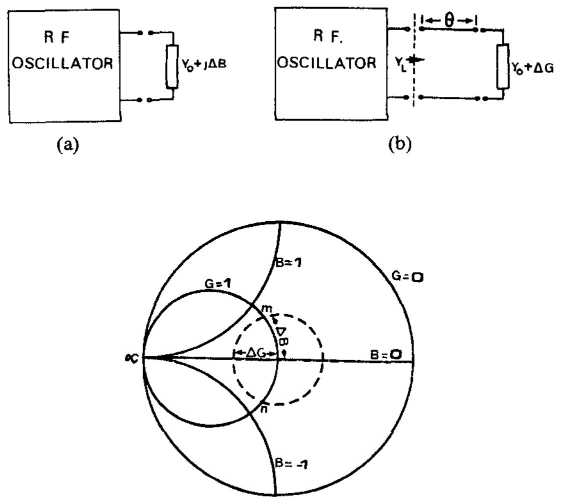

Fig. 1. Simulation of load variation.

and

$$\Delta B_L + \frac{dB_{T0}}{d\omega} \Delta\omega + \frac{dB_{T0}}{dV} V = 0 \tag{5}$$

where

$$\Delta Y_L = \Delta G_L + \Delta B_L$$

and

$$Y_{T0} = G_{T0} + B_{T0}.$$

From (4) and (5)

$$\Delta\omega = \frac{\Delta G_L \cdot \frac{dB_{T0}}{dV}}{\frac{dG_{T0}}{dV} \cdot \frac{dB_{T0}}{d\omega} - \frac{dG_{T0}}{d\omega} \cdot \frac{dB_{T0}}{dV}} - \frac{\Delta B_L \cdot \frac{dG_{T0}}{dV}}{\frac{dG_{T0}}{dV} \frac{dB_{T0}}{d\omega} - \frac{dG_{T0}}{d\omega} \cdot \frac{dB_{T0}}{dV}} \tag{6}$$

and

$$\Delta V = \frac{\Delta B_L \cdot \frac{dG_{T0}}{d\omega}}{\frac{dG_{T0}}{dV} \cdot \frac{dB_{T0}}{d\omega} - \frac{dG_{T0}}{d\omega} \cdot \frac{dB_{T0}}{dV}} - \frac{\Delta G_L \cdot \frac{dB_{T0}}{d\omega}}{\frac{dG_{T0}}{dV} \cdot \frac{dB_{T0}}{d\omega} - \frac{dG_{T0}}{d\omega} \cdot \frac{dB_{T0}}{dV}}. \tag{7}$$

III. Load Variation Simulation

From Fig. 1 it can be noted that any reactive load perturbation of value $j\Delta B$ can be represented by a nonreactive load perturbation of ΔG by suitably selecting the reference plane in the output line between the oscillator and the perturbation. For the purposes of the exact derivation of the pulling figure, we simulate the load perturbation by ΔG (Fig. 1(b)) with the transmission-line length l variable between 0 and $\lambda/2$.

For any value of $\theta = \beta l$, the transferred load admittance Y_L at

0018-9480/82/0700-1109$00.75 ©1982 IEEE